

B.Sc Part I (Hons)  
Roots and Co-efficients.

Example.

Q → Find the condition that the roots of the equation  
 $ax^3 + 3bx^2 + 3cx + d = 0$  are in (a) A.P. (b) G.P.  
(c) H.P.

Ans: → Let  $\alpha - \beta, \alpha, \alpha + \beta$  (in A.P.) be three roots of the given equation

$$\therefore \text{Sum of roots} = \alpha - \beta + \alpha + \alpha + \beta = -\frac{3b}{a}$$

$$\Rightarrow 3\alpha = -\frac{3b}{a} \Rightarrow \alpha = -\frac{b}{a}$$

Since  $\alpha$  is a root of the given equation.

$$\therefore a\alpha^3 + 3b\alpha^2 + 3c\alpha + d = 0$$

Putting  $\alpha = -\frac{b}{a}$ , we get

$$a\left(-\frac{b}{a}\right)^3 + 3b\left(-\frac{b}{a}\right)^2 + 3c\left(-\frac{b}{a}\right) + d = 0$$

$$\Rightarrow -a\frac{b^3}{a^3} + 3b\frac{b^2}{a^2} - \frac{3bc}{a} + d = 0$$

$$\Rightarrow -\frac{b^3}{a^2} + \frac{3b^3}{a^2} - \frac{3bc}{a} + d = 0$$

$$\Rightarrow \frac{2b^3}{a^2} - \frac{3bc}{a} + d = 0 \Rightarrow 2b^3 - 3abc + a^2d = 0$$

(ii) Let  $\alpha, \beta, \gamma$  are the roots of the given equation.  
Where  $\alpha, \beta, \gamma$  are in G.P.

$$\text{Let } \alpha = \frac{p}{r}, \beta = p \text{ and } \gamma = pr$$

$$\text{Product of roots } \alpha\beta\gamma = -\frac{d}{a}$$

$$\Rightarrow \frac{p}{r} \cdot p \cdot pr = -\frac{d}{a}$$

$$\Rightarrow p^3 = -\frac{d}{a} \quad \text{--- (i)}$$

$\therefore p = \beta$  is a root of the given equation so we have

$$ap^3 + 3bp^2 + 3cp + d = 0$$

$$\Rightarrow a\left(-\frac{d}{a}\right) + 3bp^2 + 3cp + d = 0 \quad \left[ p^3 = -\frac{d}{a} \right]$$

$$\Rightarrow -d + 3bp^2 + 3cp + d = 0 \Rightarrow 3bp^2 + 3cp = 0$$

$$\Rightarrow 3p(bp + c) = 0 \quad [ \because 3p \neq 0 ]$$

$$\therefore bp = -c \quad \Rightarrow b^3 p^3 = -c^3$$

$$b^3 \left(-\frac{d}{a}\right) = -c^3 \quad \left[ \because p^3 = -\frac{d}{a} \right]$$

$$\Rightarrow b^3 d - c^3 a = 0$$

(ii) Let  $\alpha, \beta, \gamma$  (in t.p) are three roots of given equation

$$\therefore \alpha + \beta + \gamma = -3b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3c/a$$

$$\text{and } \alpha\beta\gamma = -d/a$$

————— (1)

When  $\alpha, \beta, \gamma$  are in t.p then

$$\frac{1}{\alpha} + \frac{1}{\gamma} = \frac{2}{\beta}$$

$$\Rightarrow \alpha\beta + \beta\gamma = 2\gamma\alpha \Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = 3\gamma\alpha$$

$$\Rightarrow \frac{3c}{a} = 3\gamma\alpha \quad (\text{using (1)}) \Rightarrow \gamma\alpha = c/a$$

$$\text{By (1)} \quad \alpha\beta\gamma = -d/a \Rightarrow \frac{c}{\alpha}\beta = -d/a$$

$$\Rightarrow \beta = -d/c$$

As  $\beta$  is the root of the given equation then  $a\beta^3 + 3b\beta^2 + 3c\beta + d = 0$

$$a\left(-\frac{d}{c}\right)^3 + 3b\left(\frac{d}{c}\right)^2 + 3c\left(-\frac{d}{c}\right) + d = 0 \quad = 0$$

$$\Rightarrow -a\frac{d^3}{c^3} + 3b\frac{d^2}{c^2} - 3d + d = 0$$

$$\Rightarrow -a\frac{d^3}{c^3} + \frac{3bd^2}{c^2} - 2d = 0$$

$$\Rightarrow -d\left(ad^2 - \frac{3bcd}{c^2} + 2c^3\right) = 0$$

$$\Rightarrow ad^2 - 3bcd + 2c^3 = 0 \quad [ \because d \neq 0 ]$$

⇒ Anjane Kumar Singh.  
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